<u>Summer Review Packet for AP CALCULUS</u>

Directions: Complete the following problems. All work must be shown to receive full credit.

Simplify by factoring

1. $2x^{-\frac{1}{2}} + 3x^{\frac{5}{2}}$

2. $3(x+1)^{\frac{1}{2}}(2x-3)^{\frac{5}{2}} + 7(x+1)^{\frac{3}{2}}(2x-3)^{\frac{3}{2}}$

3. $(x+2)^{1/2} + x(x+2)^{-1/2}$

4. $(2x-5)^{-3/4}(x+2) - (2x-5)^{1/4}$

Exponential and Logarithm Practice

Solve each equation. Use laws of logarithms.

1. $\log 5x = \log(2x+9)$

3. $10^{2x} = 46$

4. $3e^{5x} = 18$

5. $\log(x+21) + \log x = 2$

6. $-6\log_3(x-3) = -24$

Graphs, Transformations and Domain

1. Match the name & equation to the graph.

a. $y = x$

b. $y = x^3$

c. $y = \sqrt[3]{x}$

d. $y = \dfrac{1}{x}$

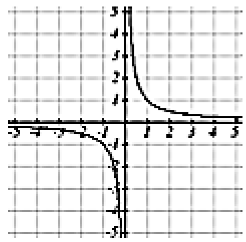
e. $y = x^2$

f. $y = \sqrt{x}$

g. $y = |x|$

h. $y = \dfrac{1}{x^2}$

i.



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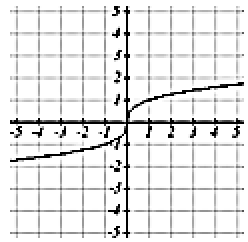

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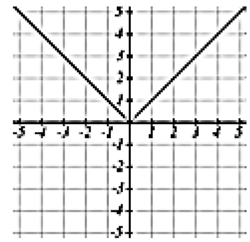

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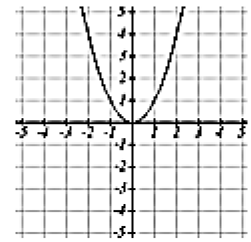

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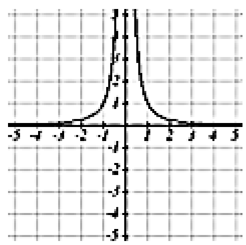

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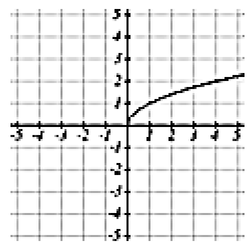

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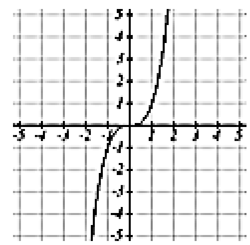

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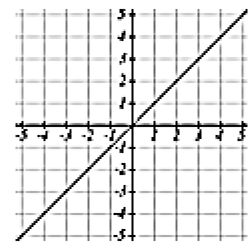


2. Match the description of the transformation with the equation.

	Description	Function
________	1. Shift to the left 1 unit	a. $y = f(-x)$
________	2. Shift to the right 1 unit	b. $y = 2f(x)$
________	3. Shift up 1 unit	c. $y = f(x+1)$
________	4. Shift down 1 unit	d. $y = \frac{1}{2}f(x)$
________	5. Makes the graph wider	e. $y = f(x) + 1$
________	6. Makes the graph more narrow	f. $y = f(x-1)$
________	7. Reflect over the x-axis	g. $y = f(x) - 1$
________	8. Reflect over the y-axis	h. $y = -f(x)$

3. Find the domain of each function.

a. $f(x) = \ln x$

b. $f(x) = \sqrt{9 - 2x}$

c. $g(x) = \frac{x}{x^2 - 16}$

d. $h(x) = \frac{5}{\sqrt{x^2 - 4}}$

Limits:

Find each of the following limits analytically:

1. $\lim_{x \to 5} \frac{2x^2 - 5x - 25}{x - 5}$

2. $\lim_{x \to 16} \frac{x - 16}{\sqrt{x} - 4}$

3. $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

4. $\lim_{x \to \infty} \frac{3x^2}{4x^2 + 2x - 1}$

5. Discuss the continuity of $(x) = \begin{cases} 6+3x & x<-2 \\ x^2-4 & x \geq -2 \end{cases}$. (Use the definition of continuity)

6. Given the function ***f*** defined by $f(x) = -\frac{x-1}{x^2+2x-3}$

a. For what values of ***x*** is ***f(x)*** discontinuous. Classify the discontinuity as removable, infinite, or jump.

b. At each point of discontinuity found in part (a) determine whether ***f(x)*** has a limit and, if so, give the value of the limit.

Derivative Practice

Find the first derivative for each of the following.

1. $y = \sin^3\left(5x^2\right)$

2. $y = (x^2+3)(x^3+4)$

3. $y = 3x^{\frac{1}{2}} - 5\sqrt[3]{x} + \pi$

4. $f(x) = \frac{2x}{\sqrt{3+x^2}}$

5. $y=(2x^3+1)^2(x-5)^4$

6. $f(x) = -2\cos x + \tan^2 x$

7. $y=x^2\sin x$

8. $y=\left(\frac{2x}{1-x}\right)^4$

Tangent Lines

1. Write an equation of the line tangent to the graph of $y=\cos(2x)$ at $x=\frac{\pi}{4}$.

2. Find $f(4)$ and $f'(4)$ if the tangent line to the graph of $f(x)$ at $x=4$ has equation $y=3x-14$.

Calculate the second derivative.

1. $y=12x^3-5x^2+3x$

2. $y=\sqrt{2x+3}$

Compute $\frac{dy}{dx}$: $y = xy^2 + 2x^2$

Find all critical points of the function.

1. $f(x) = x^3 - \frac{9}{2}x^2 - 54x + 2$

Find the absolute extrema of the function on the given interval.

1. $y = 2x^2 - 4x + 2$ $[0, 3]$

Verify Rolle's Theorem for the given interval

1. $f(x) = x + x^{-1}$, $\left[\frac{1}{2}, 2\right]$

Find a point c satisfying the conclusion of the Mean Value Theorem for the given function and interval.

1. $y = \sqrt{x}$, $[4, 9]$

Find the intervals of increase and decrease and relative extrema for the given function.

1. $y = x^3 - 6x^2$

Determine the intervals on which the function is concave up or down and find the points of inflection.

1. $y = x - 2\cos x \quad 0 \le x \le 2\pi$
2. $y = 4x^5 - 5x^4$

Related Rates:

1. Water pours into a conical tank of height 10ft and diameter of 8ft at a rate of $10\ ft^3/\text{min}$. How fast is the water level rising when it is 5 ft high?

Graphing and Derivatives

1. Each graph in Figure 2 shows the graph of a function $f(x)$ and its derivative $f'(x)$. Determine which is the function and which is the derivative.

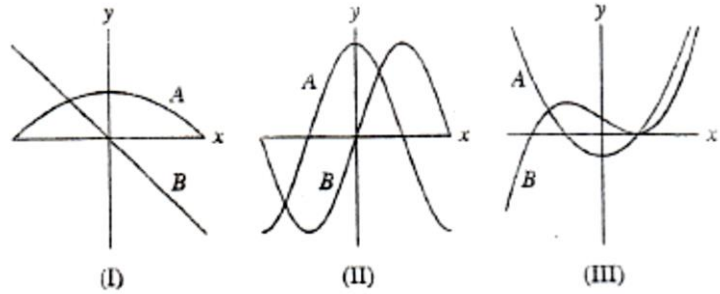
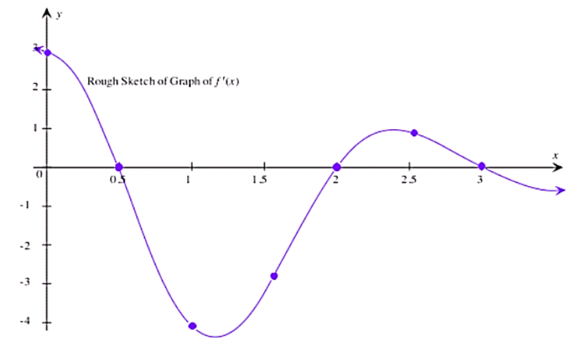


FIGURE 2 Graph of $f(x)$.

2. The figure shows the graph of the derivative, $f'(x)$ on $[0, \infty]$.
   a. Locate the points of inflection of $f(x)$ and the points where the relative maxima and minima occur.

   b. Determine the intervals on which $f(x)$ has the following properties:
      i. Increasing
      ii. Decreasing
      iii. Concave up
      iv. Concave Down



3. Match the description of $f(x)$ with the graph of its derivative $f'(x)$ in figure 1.
   a. $f(x)$ is increasing and concave up.
   b. $f(x)$ is decreasing and concave up.
   c. $f(x)$ is increasing and concave down.

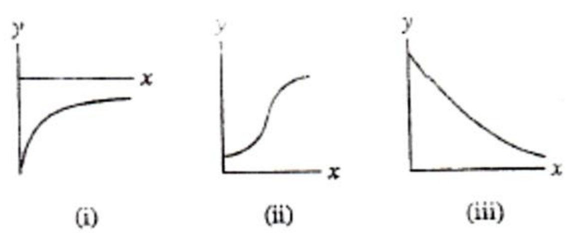


FIGURE 1 Graphs of the derivative.

PROBLEM SOLVING STRATEGY: Optimization

The strategy consists of two Big Stages. The first does not involve Calculus at all; the second is identical to what you did for max/min problems.

Stage I: Develop the function.

Your first job is to develop a function that represents the quantity you want to optimize. It can depend on only *one* variable. The steps:

1. **Draw a picture** of the physical situation.
   Also note any physical restrictions determined by the physical situation.

2. **Write an equation** that relates the quantity you want to optimize in terms of the relevant variables.

3. If necessary, use other given information to **rewrite your equation in terms of a single variable.**

Stage II: Maximize or minimize the function.

You now have a standard max/min problem to solve.

4. **Take the derivative** of your equation with respect to your single variable. Then find the critical points.

5. **Determine the maxima and minima as necessary.**
   Remember to **check the endpoints** if there are any.

6. **Justify your maxima or minima** either by reasoning about the physical situation, or with the first derivative test, or with the second derivative test.

7. Finally, **check to make sure you have answered the question as asked**: Re-read the problem and verify that you are providing the value(s) requested: an x or y value; or coordinates; or a maximum area; or a shortest time; whatever was asked.

1. An open rectangular box with square base is to be made from 48 $ft.^2$ of material. What dimensions will result in a box with the largest possible volume?

2. A gardener wants to make a rectangular enclosure using a wall as one side and 120 m of fencing for the other three sides. Find the dimensions of the garden so the gardener maximizes the area.